DEPARTMENT OF MATHEMATICS

B.Sc. (Hons) MATHEMATICS

Category-I

DISCIPLINE SPECIFIC CORE COURSE -7: GROUP THEORY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Group Theory	4	3	1	0	Class XII pass with Mathematics	Algebra

Learning Objectives

The primary objective of this course is to introduce:

- Symmetric groups, normal subgroups, factor groups, and direct products of groups.
- The notions of group homomorphism to study the isomorphism theorems with applications.
- Classification of groups with small order according to isomorphisms.

Learning Outcomes

This course will enable the students to:

- Analyse the structure of 'small' finite groups, and examine examples arising as groups of permutations of a set, symmetries of regular polygons.
- Understand the significance of the notion of cosets, Lagrange's theorem and its consequences.
- Know about group homomorphisms and isomorphisms and to relate groups using these mappings.
- Express a finite abelian group as the direct product of cyclic groups of prime power orders.
- Learn about external direct products and its applications to data security and electric circuits.

SYLLABUS OF DSC - 7

Unit – 1 **(18 hours)**

Permutation Groups, Lagrange's Theorem and Normal Subgroups

Permutation groups and group of symmetries, Cycle notation for permutations and properties, Even and odd permutations, Alternating groups; Cosets and its properties, Lagrange's theorem and consequences including Fermat's Little theorem, Number of elements in product of two finite subgroups; Normal subgroups, Factor groups, Cauchy's theorem for finite Abelian groups.

Unit – 2 **(15 hours)**

Group Homomorphisms and Automorphisms

Group homomorphisms, isomorphisms and properties, Cayley's theorem; First, Second and Third isomorphism theorems for groups; Automorphism, Inner automorphism, Automorphism groups, Automorphism groups of cyclic groups, Applications of factor groups to automorphism groups.

Unit – 3 **(12 hours)**
Direct Products of Groups and Fundamental Theorem of Finite Abelian Groups
External direct products of groups and its properties, The group of units modulo n as an external direct product, Applications to data security and electric circuits; Internal direct products; Fundamental theorem of finite abelian groups and its isomorphism classes.

Essential Reading
1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint 2021.

Suggestive Readings
- Artin, Michael. (1991). Algebra (2nd ed.). Pearson Education. Indian Reprint 2015.
- Dummit, David S., & Foote, Richard M. (2016). Abstract Algebra (3rd ed.). Student Edition. Wiley India.
- Herstein, I. N. (1975). Topics in Algebra (2nd ed.). Wiley India, Reprint 2022.
- Rotman, Joseph J. (1995). An Introduction to The Theory of Groups (4th ed.). Springer-Verlag, New York.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

DISCIPLINE SPECIFIC CORE COURSE -8: RIEMANN INTEGRATION

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Riemann Integration	4	3	1	0	Class XII pass with Mathematics	Elementary Real Analysis, and Calculus

Learning Objectives
The primary objective of this course is to:
- Understand the integration of bounded functions on a closed and bounded interval and its extension to the cases where either the interval of integration is infinite, or the integrand has infinite limits at a finite number of points on the interval of integration.
- Learn some of the properties of Riemann integrable functions, its generalization and the applications of the fundamental theorems of integration.
- Get an exposure to the utility of integration for practical purposes.

Learning Outcomes
This course will enable the students to:

- Learn about some of the classes and properties of Riemann integrable functions, and the applications of the Riemann sums to the volume and surface of a solid of revolution.
- Get insight of integration by substitution and integration by parts.
- Know about convergence of improper integrals including, beta and gamma functions.

SYLLABUS OF DSC - 8

Unit – 1 **(18 hours)**
The Riemann Integral
Definition of upper and lower Darboux sums, Darboux integral, Inequalities for upper and lower Darboux sums, Necessary and sufficient conditions for the Darboux integrability; Riemann's definition of integrability by Riemann sum and the equivalence of Riemann's and Darboux's definitions of integrability; Definition and examples of the Riemann-Stieltjes integral.

Unit – 2 **(15 hours)**
Properties of The Riemann Integral and Fundamental Theorems
Riemann integrability of monotone functions and continuous functions, Properties of Riemann integrable functions; Definitions of piecewise continuous and piecewise monotone functions and their Riemann integrability; Intermediate value theorem for integrals, Fundamental Theorems of Calculus (I and II).

Unit – 3 **(12 hours)**
Applications of Integrals and Improper Integrals
Methods of integration: integration by substitution and integration by parts; Volume by slicing and cylindrical shells, Length of a curve in the plane and the area of surfaces of revolution. Improper integrals of Type-I, Type-II and mixed type, Convergence of improper integrals, The beta and gamma functions and their properties.

Essential Readings

1. Ross, Kenneth A. (2013). Elementary Analysis: The Theory of Calculus (2nd ed.). Undergraduate Texts in Mathematics, Springer.
2. Anton, Howard, Bivens Irl and Davis Stephens (2012). Calculus (10th edn.). John Wiley & Sons, Inc.
3. Denlinger, Charles G. (2011). Elements of Real Analysis, Jones & Bartlett India Pvt. Ltd., Indian Reprint.
4. Ghorpade, Sudhir R. and Limaye, B. V. (2006). A Course in Calculus and Real Analysis. Undergraduate Texts in Mathematics, Springer (SIE). Indian Reprint.

Suggestive Readings

- Bartle, Robert G., & Sherbert, Donald R. (2015). Introduction to Real Analysis (4th ed.). Wiley, Indian Edition.
- Kumar Ajit and Kumaresan S. (2014). A Basic Course in Real Analysis. CRC Press, Taylor & Francis Group, Special Indian Edition.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

DISCIPLINE SPECIFIC CORE COURSE– 9: DISCRETE MATHEMATICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Discrete Mathematics	4	3	0	1	Class XII pass with Mathematics	Algebra and Linear Algebra

Learning Objectives

The primary objective of the course is to:

- Make students embark upon a journey of enlightenment, starting from the abstract concepts in mathematics to practical applications of those concepts in real life.
- Make the students familiar with the notion of partially ordered set and a level up with the study of lattice, Boolean algebra and related concepts.
- Culminate the journey of learning with practical applications using the knowledge attained from the abstract concepts learnt in the course.

Learning Outcomes

This course will enable the students to:

- Understand the notion of partially ordered set, lattice, Boolean algebra with applications.
- Handle the practical aspect of minimization of switching circuits to a great extent with the methods discussed in this course.
- Apply the knowledge of Boolean algebras to logic, set theory and probability theory.

SYLLABUS OF DSC - 9

Unit – 1 **(15 hours)**

Cardinality and Partially Ordered Sets

The cardinality of a set; Definitions, examples and basic properties of partially ordered sets, Order-isomorphisms, Covering relations, Hasse diagrams, Dual of an ordered set, Duality principle, Bottom and top elements, Maximal and minimal elements, Zorn's lemma, Building new ordered sets, Maps between ordered sets.

Unit – 2 **(15 hours)**

Lattices

Lattices as ordered sets, Lattices as algebraic structures, Sublattices, Products, Lattice isomorphism; Definitions, examples and properties of modular and distributive lattices; The $M_3 - N_5$ theorem with applications, Complemented lattice, Relatively complemented lattice, Sectionally complemented lattice.

Unit – 3 **(15 hours)**

Boolean Algebras and Applications

Boolean algebras, De Morgan's laws, Boolean homomorphism, Representation theorem, Boolean polynomials, Boolean polynomial functions, Equivalence of Boolean polynomials, Disjunctive normal form and conjunctive normal form of Boolean polynomials; Minimal forms of Boolean polynomials, Quine-McCluskey method, Karnaugh diagrams, Switching circuits and applications, Applications of Boolean algebras to logic, set theory and probability theory.

Practical (30 hours):

Practical/Lab work to be performed in a computer Lab using any of the Computer Algebra System Software such as Mathematica/MATLAB /Maple/Maxima/Scilab/SageMath etc., for the following problems based on:

1) Expressing relations as ordered pairs and creating relations.
2) Finding whether or not, a given relation is:
    i. Reflexive ii. Antisymmetric iii. Transitive iv. Partial order
3) Finding the following for a given partially ordered set
    i. Covering relations.
    ii. The corresponding Hasse diagram representation.
    iii. Minimal and maximal elements.
4) Finding the following for a subset *S* of a given partially ordered set *P*
    i. Whether a given element in *P* is an upper bound (lower bound) of *S* or not.
    ii. Set of all upper bounds (lower bounds) of *S*.
    iii. The least upper bound (greatest lower bound) of *S*, if it exists.
5) Creating lattices and determining whether or not, a given partially ordered set is a lattice.
6) Finding the following for a given Boolean polynomial function:
    i. Representation of Boolean polynomial function and finding its value when the Boolean variables in it take particular values over the Boolean algebra $\{0,1\}$.
    ii. Display in table form of all possible values of Boolean polynomial function over the Boolean algebra $\{0,1\}$.
7) Finding the following:
    i. Dual of a given Boolean polynomial/expression.
    ii. Whether or not two given Boolean polynomials are equivalent.
    iii. Disjunctive normal form (Conjunctive normal form) from a given Boolean expression.
    iv. Disjunctive normal form (Conjunctive normal form) when the given Boolean polynomial function is expressed by a table of values.
8) Representing a given circuit diagram (expressed using gates) in the form of Boolean expression.
9) Minimizing a given Boolean expression to find minimal expressions.

Essential Readings

1. Davey, B. A., & Priestley, H. A. (2002). Introduction to Lattices and Order (2nd ed.). Cambridge University press, Cambridge.
2. Goodaire, Edgar G., & Parmenter, Michael M. (2006). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint.
3. Lidl, Rudolf & Pilz, Gunter. (2004). Applied Abstract Algebra (2nd ed.), Undergraduate Texts in Mathematics. Springer (SIE). Indian Reprint.

Suggested Readings

- Donnellan, Thomas. (1999). Lattice Theory (1st ed.). Khosla Pub. House. Indian Reprint.
- Rosen, Kenneth H. (2019). Discrete Mathematics and its Applications (8th ed.), Indian adaptation by Kamala Krithivasan. McGraw-Hill Education. Indian Reprint 2021.